



Winter 2014 Examination

Subject & Code: Applied Maths (17301)

Model Answer

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Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
		<p><b>Important Instructions to the Examiners:</b></p> <ol style="list-style-type: none"><li>1) The Answers should be examined by key words and not as word-to-word as given in the model answer scheme.</li><li>2) The model answer and the answer written by candidate may vary but the examiner may try to assess the understanding level of the candidate.</li><li>3) The language errors such as grammatical, spelling errors should not be given more importance. (Not applicable for subject English and Communication Skills.)</li><li>4) While assessing figures, examiner may give credit for principal components indicated in the figure. The figures drawn by the candidate and those in the model answer may vary. The examiner may give credit for any equivalent figure drawn.</li><li>5) Credits may be given step wise for numerical problems. In some cases, the assumed constant values may vary and there may be some difference in the candidate's Answers and the model answer.</li><li>6) In case of some questions credit may be given by judgment on part of examiner of relevant answer based on candidate's understanding.</li><li>7) For programming language papers, credit may be given to any other program based on equivalent concept.</li></ol>		



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1)		<b>Attempt any TEN of the following:</b>		
	a)	Find the gradient of the tangent of the curve $y = \sqrt{x^3}$ at $x = 4$ .		
	Ans.	$y = \sqrt{x^3} = x^{3/2}$ $\therefore \frac{dy}{dx} = \frac{3}{2}x^{1/2}$ $\therefore$ the gradient at $x = 4$ is, $\frac{dy}{dx} = \frac{3}{2}(4)^{1/2} = 3$	1	
		<b>OR</b>		
		$y = \sqrt{x^3}$ $\therefore \frac{dy}{dx} = \frac{1}{2\sqrt{x^3}} \cdot \frac{d}{dx}(x^3)$ $= \frac{1}{2\sqrt{x^3}} \cdot 3x^2$ $\therefore$ the gradient at $x = 4$ is, $\frac{dy}{dx} = \frac{1}{2\sqrt{4^3}} \cdot 3(4)^2 = 3$	1	2
	b)	Find the radius of the curvature of the curve $y^2 = 4ax$ at the point $(a, 2a)$ .		
	Ans.	$y^2 = 4ax$ $\therefore 2y \frac{dy}{dx} = 4a$ $\therefore \frac{dy}{dx} = \frac{4a}{2y} = \frac{2a}{y}$ $\therefore \frac{d^2y}{dx^2} = -\frac{2a}{y^2} \cdot \frac{dy}{dx}$ $\therefore$ at $(a, 2a)$ , $\frac{dy}{dx} = \frac{2a}{2a} = 1$ and $\frac{d^2y}{dx^2} = -\frac{2a}{(2a)^2} \cdot 1 = -\frac{1}{2a}$ $\therefore \kappa = \frac{\frac{d^2y}{dx^2}}{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}} = \frac{-\frac{1}{2a}}{\left[1 + (1)^2\right]^{3/2}} = -\frac{1}{2a \cdot 2^{3/2}} = -\frac{1}{2^{5/2}a}$	1/2	2



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1)		$\therefore \rho = \frac{1}{\kappa} = -2^{5/2} a$ <p>OR</p> $\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}} = \frac{\left[1 + (1)^2\right]^{3/2}}{-1/2a} = -2a \cdot 2^{3/2} = -2^{5/2} a$ <p>OR</p> $y^2 = 4ax$ $\therefore 2y \frac{dy}{dx} = 4a$ $\therefore 2y \frac{d^2y}{dx^2} + 2 \frac{dy}{dx} \cdot \frac{dy}{dx} = 0 \quad \text{or} \quad 2y \frac{d^2y}{dx^2} + 2 \left(\frac{dy}{dx}\right)^2 = 0$ $\therefore \frac{d^2y}{dx^2} = -\frac{1}{y} \left(\frac{dy}{dx}\right)^2$ $\therefore \text{at } (a, 2a),$ $\frac{dy}{dx} = \frac{4a}{2 \cdot 2a} = 1 \quad \text{and} \quad \frac{d^2y}{dx^2} = -\frac{1}{2a} \cdot 1 = -\frac{1}{2a}$ $\therefore \kappa = -\frac{1}{2^{5/2} a}$ $\therefore \rho = -2^{5/2} a$	<p>1/2</p> <p>OR</p> <p>1/2</p> <p>1/2</p> <p>1/2</p> <p>1/2</p> <p>1/2</p>	<p>2</p> <p>2</p>
	c)	Evaluate $\int (\tan x + \cot x)^2 dx$		
	Ans.	$\int (\tan x + \cot x)^2 dx$ $= \int (\tan^2 x + 2 \tan x \cot x + \cot^2 x) \cdot dx$ $= \int (\tan^2 x + 2 + \cot^2 x) \cdot dx$ $= \int (\sec^2 x - 1 + 2 + \cos^2 x - 1) \cdot dx$ $= \int (\sec^2 x + \cos^2 x) \cdot dx$ $= \tan x - \cot x + c$	<p>1/2</p> <p>1/2</p> <p>1</p>	<p>2</p>
		<p><b>Note:</b> In the solution of any INDEFINITE integration problems, if the constant c is not added, 1/2 mark may be deducted.</p>		



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1)	d)	Evaluate $\int \sec^2(\log x) \frac{1}{x} dx$		
	Ans.	$\int \sec^2(\log x) \frac{1}{x} dx = \int \sec^2 t dt$ $= \tan t + c$ $= \tan(\log x) + c$ <div style="border: 1px solid black; padding: 5px; display: inline-block; margin: 10px 0;"> <math display="block">\begin{aligned} \text{Put } \log x &amp;= t \\ \therefore \frac{1}{x} dx &amp;= dt \end{aligned}</math> </div> <p style="text-align: center;"><b>OR</b></p> $\text{Put } \log x = t$ $\therefore \frac{1}{x} dx = dt$ $\therefore \int \sec^2(\log x) \frac{1}{x} dx = \int \sec^2 t dt$ $= \tan t + c$ $= \tan(\log x) + c$	$\frac{1}{2} + \frac{1}{2}$  $\frac{1}{2}$ $\frac{1}{2}$	<b>2</b>
	e)	Evaluate $\int xe^x dx$ .		
Ans.	$\int xe^x dx = x \int e^x dx - \int \left[ \int e^x dx \right] \frac{d}{dx}(x) \cdot dx$ $= xe^x - \int e^x dx$ $= xe^x - e^x + c$	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2} + \frac{1}{2}$	<b>2</b>	
f)	Evaluate $\int \frac{1}{x^2 + 3x + 2} dx$			
Ans.	$x^2 + 3x + 2 = x^2 + 3x + \frac{9}{4} - \frac{9}{4} + 2 = \left(x + \frac{3}{2}\right)^2 - \left(\frac{1}{2}\right)^2$ $\int \frac{1}{x^2 + 3x + 2} dx = \int \frac{1}{\left(x + \frac{3}{2}\right)^2 - \left(\frac{1}{2}\right)^2} dx$ $= \frac{1}{2\left(\frac{1}{2}\right)} \log \left( \frac{x + \frac{3}{2} - \frac{1}{2}}{x + \frac{3}{2} + \frac{1}{2}} \right) + c$ $= \log \left( \frac{x+1}{x+2} \right) + c$	$\frac{1}{2} + \frac{1}{2}$  $\frac{1}{2}$ $\frac{1}{2}$	<b>2</b>	



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1)		<p style="text-align: center;"><b>OR</b></p> $\int \frac{1}{x^2 + 3x + 2} dx = \int \frac{1}{x^2 + 3x + \frac{9}{4} - \frac{9}{4} + 2} dx$ $= \int \frac{1}{\left(x + \frac{3}{2}\right)^2 - \left(\frac{1}{2}\right)^2} dx$ $= \frac{1}{2\left(\frac{1}{2}\right)} \log \left( \frac{x + \frac{3}{2} - \frac{1}{2}}{x + \frac{3}{2} + \frac{1}{2}} \right) + c$ $= \log \left( \frac{x+1}{x+2} \right) + c$ <p style="text-align: center;"><b>OR</b></p> $I = \int \frac{1}{x^2 + 3x + 2} dx = \int \frac{1}{(x+1)(x+2)} dx$ $\frac{1}{(x+1)(x+2)} = \frac{A}{x+1} + \frac{B}{x+2}$ $\therefore A = 1$ $B = -1 \quad \text{-----} (*)$ $\frac{1}{(x+1)(x+2)} = \frac{1}{x+1} + \frac{-1}{x+2}$ $\therefore I = \int \left[ \frac{1}{x+1} + \frac{-1}{x+2} \right] dx$ $= \log(x+1) - \log(x+2) + c$ <p><b>Note (*):</b> There are various methods to find the values of A and B to partially factorize the given expression including direct method. Students may apply any one of the methods. Take in count all such methods.</p> <hr style="border-top: 1px dashed black;"/>	<p style="text-align: center;">1/2</p> <p style="text-align: center;">1/2</p> <p style="text-align: center;">1/2</p> <p style="text-align: center;">1/2</p> <p style="text-align: center;">1/2</p> <p style="text-align: center;">1/2</p> <p style="text-align: center;">1</p>	<p><b>2</b></p> <p><b>2</b></p> <p><b>2</b></p>
	g)	Evaluate $\int_1^2 \frac{dx}{3x-2}$		
	Ans.	$\int_1^2 \frac{dx}{3x-2} = \left[ \frac{\log(3x-2)}{3} \right]_1^2$ $= \frac{\log(6-2)}{3} - \frac{\log(3-2)}{3}$ $= \frac{\log 4}{3} - \frac{\log 1}{3}$ $= \frac{\log 4}{3}$	<p style="text-align: center;">1</p> <p style="text-align: center;">1/2</p> <p style="text-align: center;">1/2</p>	<p><b>2</b></p>



Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
1)	h)	Find the area above the x-axis bounded by $y = \sin x$ and the ordinates $x = \frac{\pi}{6}$ and $x = \frac{\pi}{3}$ .		
	Ans.	$A = \int_{\pi/6}^{\pi/3} \sin x dx$ $= [-\cos x]_{\pi/6}^{\pi/3}$ $= \left[-\cos \frac{\pi}{3}\right] - \left[-\cos \frac{\pi}{6}\right] \quad \text{---} (*)$ $= \frac{-1 + \sqrt{3}}{2} \quad \text{or} \quad 0.366$ <p style="text-align: center;"><b>OR</b></p> $A = \int_{\pi/3}^{\pi/6} \sin x dx$ $= [-\cos x]_{\pi/3}^{\pi/6}$ $= \left[-\cos \frac{\pi}{6}\right] - \left[-\cos \frac{\pi}{3}\right] \quad \text{---} (*)$ $= \frac{-\sqrt{3} + 1}{2} \quad \text{or} \quad -0.366$ $\therefore \text{Area } A = \frac{-1 + \sqrt{3}}{2} \quad \text{or} \quad 0.366$ <p><b>Note:</b> Due to the use of advance non-programmable scientific calculators, writing directly the value of the step (*) as 0.366 or -0.366 is permissible. No marks to be deducted for calculating directly the value.</p>	1/2 1/2 1 1/2 1/2 1/2	2  2
	i)	Find the order and degree of the equation		
	Ans.	$2 \frac{d^2 y}{dx^2} + \left[ 3 \sqrt{1 - \left( \frac{dy}{dx} \right)^2} - y \right] = 0$ <p><b>Order = 2</b></p> $\sqrt{1 - \left( \frac{dy}{dx} \right)^2} = \frac{1}{3} \left( y - 2 \frac{d^2 y}{dx^2} \right)$ $\therefore 1 - \left( \frac{dy}{dx} \right)^2 = \frac{1}{9} \left( y - 2 \frac{d^2 y}{dx^2} \right)^2$ <p><b>Degree = 2</b></p>	1  1	2



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1)	j)	Verify that $y = Ae^x + Be^{-x}$ is a solution of $\frac{d^2y}{dx^2} - y = 0$		
	Ans.	$y = Ae^x + Be^{-x}$ $\therefore \frac{dy}{dx} = Ae^x + Be^{-x} \cdot (-1) = Ae^x - Be^{-x}$ $\therefore \frac{d^2y}{dx^2} = Ae^x - Be^{-x} \cdot (-1)$ $= Ae^x + Be^{-x}$ $\therefore \frac{d^2y}{dx^2} = y$ $\therefore \frac{d^2y}{dx^2} - y = 0$	1/2 1/2 1/2 1/2	2
	k)	A bag contains 7 white balls, 5 black balls, and 4 red balls. If two balls are drawn at random from the bag, find the probability that both the balls are white.		
	Ans.	Total = 7 + 5 + 4 = 16 $n = {}^{16}C_2 = 120$ $\therefore m = {}^7C_2 = 21$ $\therefore p = \frac{m}{n} = \frac{21}{120} = \frac{7}{40}$ or 0.175	1/2 1/2 1	2
		<b>OR</b>		
		Total = 7 + 5 + 4 = 16 $\therefore p = \frac{{}^7C_2}{{}^{16}C_2}$ $= \frac{21}{120} = \frac{7}{40}$ or 0.175	1 1	2
		<b>Note:</b> Due to the use of advance non-programmable scientific calculators which is permissible in the board examination, writing directly the values of ${}^nC_r$ or ${}^nC_r p^r q^{n-r}$ is permissible. No marks to be deducted for calculating directly the value.		







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2)	b)	A beam is bent in the form of the curve $y = 2 \sin x - \sin 2x$ , find the radius of curvature of the beam at this point $x = \frac{\pi}{2}$ .		
	Ans.	$y = 2 \sin x - \sin 2x$ $\therefore \frac{dy}{dx} = 2 \cos x - 2 \cos 2x$ & $\frac{d^2y}{dx^2} = -2 \sin x + 4 \sin 2x$ $\therefore$ at $x = \frac{\pi}{2}$ , $\frac{dy}{dx} = 2 \cos\left(\frac{\pi}{2}\right) - 2 \cos 2\left(\frac{\pi}{2}\right) = 2$ and $\frac{d^2y}{dx^2} = -2 \sin\left(\frac{\pi}{2}\right) + 4 \sin 2\left(\frac{\pi}{2}\right) = -2$ $\therefore \rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}} = \frac{\left[1 + (2)^2\right]^{\frac{3}{2}}}{-2} = -5.590$ OR $\kappa = \frac{\frac{d^2y}{dx^2}}{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}} = \frac{-2}{\left[1 + (2)^2\right]^{\frac{3}{2}}} = -0.1789$ $\therefore \rho = \frac{1}{\kappa} = -5.590$	1 1  1/2 1/2  1  OR 1/2  1/2	4
	c)	A metal wire 36 cm long is bent to form a rectangle. Find its dimensions when its area is maximum.		
	Ans.	Let x and y be the sides of rectangle. $\therefore 2x + 2y = 36$ or $x + y = 18$ $\therefore y = 18 - x$ But area $A = xy = x(18 - x) = 18x - x^2$ $\therefore \frac{dA}{dx} = 18 - 2x$	1/2  1 1/2	



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2)		$\therefore \frac{d^2A}{dx^2} = -2$ <p>For stationary values, <math>\frac{dA}{dx} = 0</math></p> $\therefore 18 - 2x = 0$ $\therefore x = 9$ <p>At <math>x = 9</math>, <math>\frac{d^2A}{dx^2} = -2 &lt; 0</math></p> <p><math>\therefore</math> At <math>x = 9</math>, A has maximum value and the other side is</p> $y = 18 - x = 9$	<p>1/2</p> <p>1/2</p> <p>1/2</p> <p>1/2</p>	4
	d)	Evaluate $\int \frac{x-3}{x^3-3x^2-16x+48} dx$		
	Ans.	$\int \frac{x-3}{x^3-3x^2-16x+48} dx = \int \frac{x-3}{(x-3)(x^2-16)} dx$ $= \int \frac{1}{x^2-16} dx$ $= \int \frac{1}{x^2-4^2} dx$ $= \frac{1}{8} \log \left( \frac{x-4}{x+4} \right) + c$ <p style="text-align: center;"><b>OR</b></p> $\int \frac{x-3}{x^3-3x^2-16x+48} dx = \int \frac{x-3}{(x-3)(x-4)(x+4)} dx$ $= \int \frac{1}{(x-4)(x+4)} dx$ $= \int \left[ \frac{1/8}{x-4} + \frac{-1/8}{x+4} \right] dx$ $= \frac{1}{8} \log(x-4) - \frac{1}{8} \log(x+4) + c$	<p>1</p> <p>1</p> <p>2</p>	4
	e)	Evaluate $\int \frac{1}{x[9+(\log x)^2]} dx$		
	Ans.	<p>Put <math>\log x = t</math></p> $\therefore \frac{1}{x} dx = dt$	1	



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2)		$\therefore \int \frac{1}{x[9+(\log x)^2]} dx = \int \frac{1}{9+t^2} dt$ $= \int \frac{1}{3^2+t^2} dt$ $= \frac{1}{3} \tan^{-1}\left(\frac{t}{3}\right) + c$ $= \frac{1}{3} \tan^{-1}\left(\frac{\log x}{3}\right) + c$ <p style="text-align: center;"><b>OR</b></p> $\therefore \int \frac{1}{x[9+(\log x)^2]} dx \quad \left  \begin{array}{l} \text{Put } \log x = t \\ \therefore \frac{1}{x} dx = dt \end{array} \right.$ $= \int \frac{1}{9+t^2} dt$ $= \int \frac{1}{3^2+t^2} dt$ $= \frac{1}{3} \tan^{-1}\left(\frac{t}{3}\right) + c$ $= \frac{1}{3} \tan^{-1}\left(\frac{\log x}{3}\right) + c$	<p>1/2</p> <p>1/2</p> <p>1</p> <p>1</p> <p>1</p>	4
	f)	<p>Evaluate <math>\int \frac{\sec^2 x}{(1+\tan x)(3+\tan x)} dx</math></p> $\int \frac{\sec^2 x}{(1+\tan x)(3+\tan x)} dx \quad \left  \begin{array}{l} \text{Put } \tan x = t \\ \therefore \sec^2 x dx = dt \end{array} \right.$ $= \int \frac{1}{(1+t)(3+t)} dt$ $\therefore \frac{1}{(1+t)(3+t)} = \frac{A}{1+t} + \frac{B}{3+t}$ $\therefore A = \frac{1}{2}$ $B = -\frac{1}{2}$	<p>1</p> <p>1</p> <p>1</p>	4
<b>[Please refer the note written in the question 1 (f).]</b>				



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2)		$\therefore \frac{1}{(1+t)(3+t)} = \frac{1}{1+t} + \frac{-1}{3+t}$ $\int \frac{1}{(1+t)(3+t)} dt = \int \left[ \frac{1}{1+t} + \frac{-1}{3+t} \right] dx$ $= \frac{1}{2} \log[1+t] - \frac{1}{2} \log[3+t] + c$ $= \frac{1}{2} \log[1+\tan x] - \frac{1}{2} \log[3+\tan x] + c$	1/2 1/2	4
3)	a)	<p><b>Attempt any four.</b></p> <p>Evaluate <math>\int_0^{\pi/4} x \sec^2 x dx</math></p>		
	Ans.	$\int_0^{\pi/4} x \sec^2 x dx = \left[ x \int \sec^2 x dx - \int \left( \int \sec^2 x dx \right) \frac{d}{dx}(x) dx \right]_0^{\pi/4}$ $= \left[ x \tan x - \int \tan x dx \right]_0^{\pi/4}$ $= \left[ x \tan x - \log(\sec x) \right]_0^{\pi/4}$ $= \left[ \frac{\pi}{4} \tan \frac{\pi}{4} - \log \left( \sec \frac{\pi}{4} \right) \right] - [0 - \log(\sec 0)]$ $= \frac{\pi}{4} - \log \sqrt{2} \quad \text{or} \quad 0.439$	1 1 1 1/2 1/2	4
		<p><b>Note:</b> In case of definite integrations, the problem may be solved by without limits and then the limits would be applied, as illustrated below:</p>		
		$\int x \sec^2 x dx = x \int \sec^2 x dx - \int \left( \int \sec^2 x dx \right) \frac{d}{dx}(x) dx$ $= x \tan x - \int \tan x dx$ $= x \tan x - \log(\sec x)$	1 1 1	
		$\int_0^{\pi/4} x \sec^2 x dx = \left[ x \tan x - \log(\sec x) \right]_0^{\pi/4}$ $= \left[ \frac{\pi}{4} \tan \frac{\pi}{4} - \log \left( \sec \frac{\pi}{4} \right) \right] - [0 - \log(\sec 0)]$ $= \frac{\pi}{4} - \log \sqrt{2} \quad \text{or} \quad 0.439$	1/2 1/2	4



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3)	b)	<p>Evaluate <math>\int_1^2 \frac{\sqrt{x}}{\sqrt{x} + \sqrt{3-x}} dx</math></p> $I = \int_1^2 \frac{\sqrt{x}}{\sqrt{x} + \sqrt{3-x}} dx$ <p style="text-align: right;">Re place <math>x \rightarrow 3-x</math> <math>\therefore 3-x \rightarrow x</math></p> <hr style="width: 20%; margin-left: auto; margin-right: 0;"/> $\therefore I = \int_1^2 \frac{\sqrt{3-x}}{\sqrt{3-x} + \sqrt{x}} dx$ $\therefore 2I = \int_1^2 \frac{\sqrt{x} + \sqrt{3-x}}{\sqrt{x} + \sqrt{3-x}} dx$ $\therefore 2I = \int_1^2 1 \cdot dx$ $\therefore 2I = [x]_1^2$ $\therefore 2I = 2-1$ $\therefore I = \frac{1}{2}$ <p style="text-align: center;"><b>OR</b></p> $I = \int_1^2 \frac{\sqrt{x}}{\sqrt{x} + \sqrt{3-x}} dx$ $= \int_1^2 \frac{\sqrt{3-x}}{\sqrt{3-x} + \sqrt{3-(3-x)}} dx$ $\therefore I = \int_1^2 \frac{\sqrt{3-x}}{\sqrt{3-x} + \sqrt{x}} dx$ $\therefore 2I = \int_1^2 \frac{\sqrt{x} + \sqrt{3-x}}{\sqrt{x} + \sqrt{3-x}} dx$ $\therefore 2I = \int_1^2 1 \cdot dx$ $\therefore 2I = [x]_1^2$ $\therefore 2I = 2-1$ $\therefore I = \frac{1}{2}$ <hr style="border-top: 1px dashed black;"/>	<p>1</p> <p>1</p> <p><math>\frac{1}{2}</math></p> <p>1</p> <p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p> <p>1</p> <p><math>\frac{1}{2}</math></p> <p>1</p> <p><math>\frac{1}{2}</math></p>	<p>4</p> <p>4</p>

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3)	c)	Find the area bounded by the curve $y = x^2$ and the line $y = x$ .		
	Ans.	<p>Given <math>y = x^2, y = x</math></p> <p><math>\therefore x^2 = x</math></p> <p><math>\therefore x^2 - x = 0</math></p> <p><math>\therefore x = 0, 1</math></p> <p><math>\therefore A = \int_a^b (y_2 - y_1) dx</math></p> <p><math>= \int_0^1 (x - x^2) dx</math></p> <p><math>= \left[ \frac{x^2}{2} - \frac{x^3}{3} \right]_0^1</math></p> <p><math>= \left[ \frac{1}{2} - \frac{1}{3} \right] - [0 - 0]</math></p> <p><math>= 0.167</math></p> <p style="text-align: center;"><b>OR</b></p> <p>Given <math>y = x^2, y = x</math></p> <p><math>\therefore x^2 = x</math></p> <p><math>\therefore x^2 - x = 0</math></p> <p><math>\therefore x = 0, 1</math></p> <p><math>\therefore A = \int_a^b (y_2 - y_1) dx</math></p> <p><math>= \int_0^1 (x^2 - x) dx</math></p> <p><math>= \left[ \frac{x^3}{3} - \frac{x^2}{2} \right]_0^1</math></p> <p><math>= \left[ \frac{1}{3} - \frac{1}{2} \right] - [0 - 0]</math></p> <p><math>= -\frac{1}{6} \text{ or } -0.167</math></p> <p><math>\therefore \text{Area } A = \frac{1}{6} \text{ or } 0.167</math></p> <p>-----</p>	<p>1</p> <p>1/2</p> <p>1</p> <p>1/2</p> <p>1</p> <p><b>4</b></p> <p>OR</p> <p>1</p> <p>1/2</p> <p>1</p> <p>1/2</p> <p>1/2</p> <p>1/2</p> <p><b>4</b></p>	



Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
3)	d)	Solve $1 - 2 \frac{dy}{dx} = \cos^2(x - 2y)$		
	Ans.	$1 - 2 \frac{dy}{dx} = \cos^2(x - 2y)$ Put $x - 2y = v$ $\therefore 1 - 2 \frac{dy}{dx} = \frac{dv}{dx}$ $\therefore \frac{dv}{dx} = \cos^2 v$ $\therefore \frac{dv}{\cos^2 v} = dx$ $\therefore \int \sec^2 v dv = \int dx$ $\therefore \tan v = x + c$ $\therefore \tan(x - 2y) = x + c$	1 1 1/2 1 1/2	4
	e)	Evaluate $\frac{dy}{dx} = \frac{y}{x} + \sin \frac{y}{x}$		
	Ans.	$\frac{dy}{dx} = \frac{y}{x} + \sin \frac{y}{x}$ Put $\frac{y}{x} = v$ or $y = vx$ $\therefore \frac{dy}{dx} = v + x \frac{dv}{dx}$ $\therefore v + x \frac{dv}{dx} = v + \sin v$ $\therefore x \frac{dv}{dx} = \sin v$ $\therefore \cos ec v dv = \frac{dx}{x}$ $\therefore \int \cos ec v dv = \int \frac{dx}{x}$ $\therefore \log(\cos ec v - \cot v) = \log x + c$ $\therefore \log\left(\cos ec \frac{y}{x} - \cot \frac{y}{x}\right) = \log x + c$	1 1/2 1/2 1/2 1/2+1/2 1/2	4



Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
3)	f)	Evaluate $(x+1)\frac{dy}{dx} - y = e^x(x+1)^2$		
	Ans.	$(x+1)\frac{dy}{dx} - y = e^x(x+1)^2$ $\therefore \frac{dy}{dx} - \frac{1}{x+1} \cdot y = e^x(x+1)$ $P = -\frac{1}{x+1} \quad \text{and} \quad Q = e^x(x+1)$ $\therefore IF = e^{\int p dx} = e^{\int -\frac{1}{x+1} dx} = e^{-\log(x+1)} = \frac{1}{x+1}$ $\therefore y \cdot IF = \int Q \cdot IF \cdot dx + c$ $\therefore y \cdot \frac{1}{x+1} = \int \frac{1}{x+1} \cdot e^x(x+1) \cdot dx$ $\therefore y \cdot \frac{1}{x+1} = \int e^x \cdot dx$ $\therefore y \cdot \frac{1}{x+1} = e^x + c$	1 1 1 1	4
4)	a)	Attempt any four. Evaluate $\int_0^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx$		
	Ans.	$I = \int_0^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx$ <div style="border: 1px solid black; padding: 5px; display: inline-block; margin-left: 20px;">                     Replace <math>x \rightarrow \pi/2 - x</math>  <math>\therefore \sin x \rightarrow \cos x</math>                      &amp; <math>\cos x \rightarrow \sin x</math> </div> $\therefore I = \int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$ $\therefore 2I = \int_0^{\pi/2} \frac{\sqrt{\cos x} + \sqrt{\sin x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx$ $\therefore 2I = \int_0^{\pi/2} 1 \cdot dx$ $\therefore 2I = [x]_0^{\pi/2}$ $= \frac{\pi}{2} - 0$ $\therefore I = \frac{\pi}{4}$	1 1 1/2 1 1/2	4





Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
4)		<p style="text-align: center;"><b>OR</b></p> $I = \int_0^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx$ $= \int_0^{\pi/2} \frac{\sqrt{\cos\left(\frac{\pi}{2} - x\right)}}{\sqrt{\cos\left(\frac{\pi}{2} - x\right)} + \sqrt{\sin\left(\frac{\pi}{2} - x\right)}} dx$ $\therefore I = \int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$ $\therefore 2I = \int_0^{\pi/2} \frac{\sqrt{\cos x} + \sqrt{\sin x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx$ $\therefore 2I = \int_0^{\pi/2} 1 \cdot dx$ $\therefore 2I = [x]_0^{\pi/2}$ $= \frac{\pi}{2} - 0$ $\therefore I = \frac{\pi}{4}$ <hr style="border-top: 1px dashed black;"/> <p>b)</p> <p>Evaluate <math>\int_0^1 x^2 \sqrt{1-x} \cdot dx</math></p>	<p style="text-align: center;">1/2</p> <p style="text-align: center;">1/2</p> <p style="text-align: center;">1</p> <p style="text-align: center;">1/2</p> <p style="text-align: center;">1</p> <p style="text-align: center;">1/2</p>	<b>4</b>
	Ans.	$I = \int_0^1 x^2 \sqrt{1-x} \cdot dx$ $\therefore I = \int_0^1 (1-x)^2 \sqrt{1-(1-x)} \cdot dx$ $= \int_0^1 (1-2x+x^2) \sqrt{x} \cdot dx$ $= \int_0^1 \left( \sqrt{x} - 2x^{3/2} + x^{5/2} \right) \cdot dx$ $= \left[ \frac{2}{3} x^{3/2} - 2 \frac{x^{5/2}}{5/2} + \frac{x^{7/2}}{7/2} \right]_0^1$ $= \left[ \frac{2}{3} x^{3/2} - \frac{4}{5} x^{5/2} + \frac{2}{7} x^{7/2} \right]_0^1$ $= \left[ \frac{2}{3} 1^{3/2} - \frac{4}{5} 1^{5/2} + \frac{2}{7} 1^{7/2} \right] - [0-0+0]$ $= \frac{16}{105} \quad \text{or} \quad 0.152$	<p style="text-align: center;">1/2</p> <p style="text-align: center;">1</p> <p style="text-align: center;">1</p> <p style="text-align: center;">1</p> <p style="text-align: center;">1/2</p>	<b>4</b>



Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
4)	c)	Find by integration the area of the circle $x^2 + y^2 = a^2$ .		
	Ans.	$x^2 + y^2 = a^2$ $\therefore y^2 = a^2 - x^2$ $\therefore y = \sqrt{a^2 - x^2}$ $\text{At } y = 0, \quad a^2 - x^2 = 0$ $\therefore x = -a, \quad a$ $\therefore A = 4 \int_a^b y dx$ $= 4 \int_0^a \sqrt{a^2 - x^2} dx$ $= 4 \left[ \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \left( \frac{x}{a} \right) \right]_0^a$ $= 4 \left[ 0 + \frac{a^2}{2} \sin^{-1}(1) \right] - \left[ 0 + \frac{a^2}{2} \sin^{-1}(0) \right]$ $= 4 \left[ \frac{a^2}{2} \cdot \frac{\pi}{2} \right]$ $= \pi a^2$ <p style="text-align: center;"><b>OR</b></p> $x = -a, \quad a$ $\therefore A = 2 \int_a^b y dx$ $= 2 \int_{-a}^a \sqrt{a^2 - x^2} dx$ $= 2 \left[ \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \left( \frac{x}{a} \right) \right]_{-a}^a$ $= 2 \left[ 0 + \frac{a^2}{2} \sin^{-1}(1) \right] - \left[ 0 + \frac{a^2}{2} \sin^{-1}(-1) \right]$ $= 2 \left[ \frac{a^2}{2} \cdot \frac{\pi}{2} + \frac{a^2}{2} \cdot \frac{\pi}{2} \right]$ $= \pi a^2$	1 1/2 1 1/2 1/2 1/2 1 1/2 1/2 1/2	4 4
	d)	Evaluate $\frac{dy}{dx} = e^{2x-3y} + 4x^2e^{-3y}$		
	Ans.	$\frac{dy}{dx} = e^{2x-3y} + 4x^2e^{-3y}$ $= e^{2x} \cdot e^{-3y} + 4x^2e^{-3y}$ $= (e^{2x} + 4x^2)e^{-3y}$	1	



Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
4)		$\therefore \frac{dy}{e^{-3y}} = (e^{2x} + 4x^2) dx$ $\therefore \int e^{3y} dy = \int (e^{2x} + 4x^2) dx$ $\therefore \frac{e^{3y}}{3} = \frac{e^{2x}}{2} + \frac{4}{3} x^3 + c$	1 1 1	4
	e)	<p>Evaluate <math>(2xy + y^2) dx + (x^2 + 2xy + \sin y) dy = 0</math></p> <p>Ans. <math>(2xy + y^2) dx + (x^2 + 2xy + \sin y) dy = 0</math>  <math>M = 2xy + y^2</math>  <math>\therefore \frac{\partial M}{\partial y} = 2x + 2y</math>  <math>N = x^2 + 2xy + \sin y</math>  <math>\therefore \frac{\partial N}{\partial x} = 2x + 2y</math>  <math>\therefore</math> the equation is exact.</p> $\int_{y \text{ constant}} M dx + \int_{\text{terms free from } x} N dy = c$ $\int (2xy + y^2) dx + \int \sin y dy = c$ $\therefore 2y \cdot \frac{x^2}{2} + y^2 x - \cos y = c$ <p>or <math>x^2 y + xy^2 - \cos y = c</math></p>	1      1  1/2  1/2   1  1	4
	f)	<p>Show that <math>y^2 = ax^2</math> is a solution of <math>x \left( \frac{dy}{dx} \right)^2 - 2y \frac{dy}{dx} + ax = 0</math>.</p> <p>Ans. <math>y^2 = ax^2</math>  <math>\therefore 2y \frac{dy}{dx} = 2ax</math>  <math>\therefore \frac{dy}{dx} = \frac{2ax}{2y} = \frac{ax}{y}</math>  <math>\therefore x \left( \frac{dy}{dx} \right)^2 - 2y \frac{dy}{dx} + ax = x \left( \frac{ax}{y} \right)^2 - 2y \cdot \frac{ax}{y} + ax</math>  <math>= \frac{a^2 x^3}{y^2} - 2ax + ax</math>  <math>= \frac{a^2 x^3}{ax^2} - 2ax + ax</math>  <math>= 0</math></p>	1       1  1  1	4



Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
5)		<p><b>Attempt any four.</b></p> <p>a) A husband and wife appear in an interview for two vacancies in the same post. The probability of husband's selection is <math>\frac{1}{7}</math> and that of wife selection is <math>\frac{1}{5}</math>. What is probability that:</p> <p>1) Both of them will be selected, 2) None of them will be selected.</p> <p>Ans. <math>P(H) = \frac{1}{7}</math>      <math>P(H') = 1 - \frac{1}{7} = \frac{6}{7}</math>      <math>\frac{1}{2}</math></p> <p><math>P(W) = \frac{1}{5}</math>      <math>P(W') = 1 - \frac{1}{5} = \frac{4}{5}</math>      <math>\frac{1}{2}</math></p> <p><math>P(\text{Both selected}) = P(H \&amp; W)</math>  <math>= P(H) \cdot P(W)</math>  <math>= \frac{1}{7} \cdot \frac{1}{5}</math>  <math>= \frac{1}{35}</math> or 0.0286      <math>\frac{1}{2}</math></p> <p><math>P(\text{None is selected}) = P(H' \&amp; W')</math>  <math>= P(H') \cdot P(W')</math>  <math>= \frac{6}{7} \cdot \frac{4}{5}</math>  <math>= \frac{24}{35}</math> or 0.686      <math>\frac{1}{2}</math></p> <p>-----</p> <p>b) The overall percentage of failures in a certain examination is 20. If six candidates appear in an examination, what is the probability that at least five pass the examination?</p> <p>Ans. % of failure = 20%  <math>\therefore</math> % of passing = 80%  <math>\therefore p = \frac{80}{100} = 0.8</math>      <math>q = 1 - 0.8 = 0.2</math>      1</p> <p><math>n = 6</math>      1</p> <p><math>\therefore p(\text{at least 5}) = p(5) + p(6)</math>  <math>= {}^6C_5 (0.8)^5 (0.2)^1 + {}^6C_6 (0.8)^6 (0.2)^0</math>  <math>= 0.6553</math>      1</p>	4	4
			1	4



Que. No.	Sub. Que.	Model Answers	Marks	Total Marks																																											
6)		<p><b>Note:</b> Due to the use of advance non-programmable scientific calculators, writing directly the values of <math>{}^n C_r</math> or <math>{}^n C_r p^r q^{n-r}</math> is permissible. No marks to be deducted for calculating directly the value.</p> <p>-----</p> <p>c)</p> <p>A skilled typist, on routine work, kept a record of mistakes per day during 300 working days. Fit a Poisson distribution to the set of observations.</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <tr><td>x</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr> <tr><td>y</td><td>143</td><td>90</td><td>42</td><td>12</td><td>9</td><td>3</td><td>1</td></tr> </table> <p>Ans.</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <tr><td>x</td><td>y</td><td>xy</td></tr> <tr><td>0</td><td>143</td><td>0</td></tr> <tr><td>1</td><td>90</td><td>90</td></tr> <tr><td>2</td><td>42</td><td>84</td></tr> <tr><td>3</td><td>12</td><td>36</td></tr> <tr><td>4</td><td>9</td><td>36</td></tr> <tr><td>5</td><td>3</td><td>15</td></tr> <tr><td>6</td><td>1</td><td>6</td></tr> <tr><td></td><td><b>300</b></td><td><b>267</b></td></tr> </table> <p><math>\therefore \text{mean } m = \frac{267}{300} = 0.89</math></p> <p><math>\therefore p = \frac{e^{-m} m^r}{r!} = \frac{e^{-0.89} (0.89)^r}{r!}</math></p> <p>-----</p> <p>d)</p> <p>Evaluate <math>\int \frac{dx}{1 + \sin x + \cos x}</math></p> <p>Ans.</p> <p>Put <math>\tan \frac{x}{2} = t</math></p> <p><math>\therefore dx = \frac{2dt}{1+t^2}</math>, <math>\sin x = \frac{2t}{1+t^2}</math>, <math>\cos x = \frac{1-t^2}{1+t^2}</math></p> <p><math>\int \frac{dx}{\sin x + \cos x + 1} = \int \frac{\frac{2dt}{1+t^2}}{1 + \frac{2t}{1+t^2} + \frac{1-t^2}{1+t^2}}</math></p> <p style="text-align: center;"><math>= \int \frac{\frac{2dt}{1+t^2}}{\frac{1+t^2 + 2t + 1 - t^2}{1+t^2}}</math></p>	x	0	1	2	3	4	5	6	y	143	90	42	12	9	3	1	x	y	xy	0	143	0	1	90	90	2	42	84	3	12	36	4	9	36	5	3	15	6	1	6		<b>300</b>	<b>267</b>	1	
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Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
5)		$= 2 \int \frac{dt}{2+2t}$ $= \int \frac{dt}{1+t}$ $= \log[1+t] + c$ $= \log \left[ 1 + \tan \frac{x}{2} \right] + c$	1/2 1 1	4
	e)	Evaluate $\int_0^1 x(1-x)^{3/2} dx$		
	Ans.	$\int_0^1 x(1-x)^{3/2} dx = \int_0^1 (1-x)[1-(1-x)]^{3/2} dx$ $= \int_0^1 (1-x)x^{3/2} dx$ $= \int_0^1 (x^{3/2} - x \cdot x^{3/2}) dx$ $= \int_0^1 (x^{3/2} - x^{5/2}) dx$ $= \left[ \frac{x^{5/2}}{5/2} - \frac{x^{7/2}}{7/2} \right]_0^1$ $= \left[ \frac{1}{5/2} - \frac{1}{7/2} \right] - 0$ $= \frac{4}{35} \text{ or } 0.114$	1/2 1/2 1/2 1/2+1/2 1 1/2	
f)	Solve $\frac{dy}{dx} = -\frac{y \cos x + \sin y + y}{\sin x + x \cos y + x}$			
	Ans.	$\frac{dy}{dx} = -\frac{y \cos x + \sin y + y}{\sin x + x \cos y + x}$ $\therefore (y \cos x + \sin y + y) dx + (\sin x + x \cos y + x) dy = 0$ $M = y \cos x + \sin y + y$ $\therefore \frac{\partial M}{\partial y} = \cos x + \cos y + 1$	1	4



Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
6)		$N = \sin x + x \cos y + x$ $\therefore \frac{\partial N}{\partial x} = \cos x + \cos y + 1$ <p><math>\therefore</math> the equation is exact.</p> $\int_{y \text{ constant}} M dx + \int \text{terms free from } x N dy = c$ $\int (y \cos x + \sin y + y) dx + \int 0 dy = c$ $\therefore y \sin x + x \sin y + xy = c$	<p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p> <p>1</p> <p>1</p>	4
6)	a)	<p><b>Attempt any four.</b></p> <p>a) A coin is tossed and a die is rolled. Show that the events head and six are independent and mutually exclusive.</p> <p>Ans. <b>Case I)</b> consider the experiment of two events "A coin is tossed and a die is rolled" are taken together.</p> $\therefore S = \{(H, 1), (H, 2), (H, 3), (H, 4), (H, 5), (H, 6), (T, 1), (T, 2), (T, 3), (T, 4), (T, 5), (T, 6)\}.$ $\therefore n = 12.$ <p>Let A = event of occurring head,</p> $\therefore A = \{(H, 1), (H, 2), (H, 3), (H, 4), (H, 5), (H, 6)\}$ $\therefore m = 6$ $\therefore P(A) = \frac{m}{n} = \frac{6}{12} = \frac{1}{2}$ <p>Let B = event of occurring six</p> $\therefore B = \{(H, 6), (T, 6)\}$ $\therefore m = 2$ $\therefore P(B) = \frac{m}{n} = \frac{2}{12} = \frac{1}{6}$ <p>Now <math>A \cap B = \{(H, 6)\}</math></p> $\therefore m = 1$ <p>The probability of happening head and six is</p> $\therefore P(A \cap B) = \frac{m}{n} = \frac{1}{12}$ <p>But <math>P(A)P(B) = \frac{1}{2} \times \frac{1}{6} = \frac{1}{12}.</math></p> $\therefore P(A \cap B) = P(A)P(B)$ <p><math>\therefore</math> the events are independent.</p>	<p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p>	



Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
6)		<p>But</p> $P(A \cap B) \neq 0 \quad [ \text{or also } P(A \cup B) = P(A) + P(B) - P(A \cap B) ]$ <p><math>\therefore</math> the events are not mutually exclusive.</p> <p><b>Case II)</b> Consider the experiment of two events "A coin is tossed and a die is rolled" are not taken together and done exclusively.</p> <p>i. the set of tossing coin is { H, T}. Consequently n = 2. Now let A = event of occurring head, then m = 1 and hence <math>P(A) = \frac{m}{n} = \frac{1}{2}</math>.</p> <p>ii. the set of rolling of die is {1, 2, 3, 4, 5, 6}. Consequently n = 6. Now let B = event of occurring six, then m = 1 in this case and hence <math>P(B) = \frac{m}{n} = \frac{1}{6}</math></p> <p>Now here in this case <math>A \cap B = \Phi</math> and hence <math>P(A \cap B) = 0</math> shows that the events are mutually exclusive but the events are not independent as:</p> $P(A)P(B) = \frac{1}{2} \times \frac{1}{6} = \frac{1}{12} \neq P(A \cap B).$ <hr style="border-top: 1px dashed black;"/>	<p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p>	<p>4</p>
	b)	<p>If A and B are two events such that <math>P(A) = \frac{1}{2}</math>, <math>P(B) = \frac{1}{3}</math> and <math>P(A \cap B) = \frac{7}{12}</math>, find <math>P(A' \cap B')</math></p>		
	Ans.	$P(A' \cap B') = P(A \cup B)'$ $= 1 - P(A \cup B)$ $= 1 - [P(A) + P(B) - P(A \cap B)]$ $= 1 - \left[ \frac{1}{2} + \frac{1}{3} - \frac{7}{12} \right]$ $= \frac{3}{4} \quad \text{or} \quad 0.75$	<p>1</p> <p>1</p> <p>1</p> <p>1</p>	<p>4</p>
		<b>OR</b>		





Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
6)		<b>OR</b>  $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ $= \frac{1}{2} + \frac{1}{3} - \frac{7}{12}$ $= \frac{1}{4} \text{ or } 0.25$ $P(A' \cap B') = P(A \cup B)'$ $= 1 - P(A \cup B)$ $= 1 - \frac{1}{4} \text{ or } 1 - 0.25$ $= \frac{3}{4} \text{ or } 0.75$	 1 1 1/2 1 1/2	4
	c)	<p>In a sample of 1000 cases the mean of a certain test is 14 and standard deviation is 2.5. Assuming the distribution to be normal, find 1) how many students score between 12 and 15? and 2) how many students score above 18?</p>		
	Ans.	<p>Given <math>\bar{x} = 14</math> <math>\sigma = 2.5</math> <math>N = 1000</math></p> <p>1) <math>z = \frac{12-14}{2.5} = -0.8</math> <math>z = \frac{15-14}{2.5} = 0.4</math></p> <p><math>\therefore P(12 \leq x \leq 15) = P(-0.8 \leq z \leq 0.4)</math></p> <p style="text-align: center;"><math>= P(-0.8 \leq z \leq 0) + P(0 \leq z \leq 0.4)</math></p> <p style="text-align: center;"><math>= P(0 \leq z \leq 0.8) + P(0 \leq z \leq 0.4)</math></p>	1/2	
		<p><b>Note:</b> To get the further solution of the problem, the students are required the set of the values of the area under standard normal curves. And this set of values is not provided with this question. These values can only be obtained from the Table of Area Under Standard Normal Curve. This table is provided at the end of the solution for the sake of your convenience. Please go through the same. The further solution is formed using the values obtained from this table only.</p>		
		<p><math>\therefore P(12 \leq x \leq 15) = 0.2881 + 0.1554</math></p> <p style="text-align: center;"><math>= 0.4435</math></p> <p><math>\therefore \text{no. of students} = N \cdot P = 1000 \times 0.4435 = 443.5 \text{ i.e., } 444</math></p>	1/2	
			1/2	



Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
6)		$2) z = \frac{18-14}{2.5} = 1.6$ $\therefore P(18 \leq x) = P(1.6 \leq z)$ $= 0.5 - P(0 \leq z \leq 1.6)$ $= 0.5 - 0.4452$ $= 0.0548$ $\therefore \text{no. of students} = N \cdot P = 1000 \times 0.0548 = 54.8 \text{ i.e., } 55$ <p><b>Note:</b> If the students have adopted any other assumptions, due credit may be given.</p>	<p>1/2</p> <p>1/2</p> <p>1/2</p> <p>1/2</p>	4
	d)	<p>Show that the equation of the tangent to the curve <math>\left(\frac{x}{a}\right)^m + \left(\frac{y}{b}\right)^m = 2</math> at the point (a, b) is <math>\frac{x}{a} + \frac{y}{b} = 2</math>.</p>		
	Ans.	$\left(\frac{x}{a}\right)^m + \left(\frac{y}{b}\right)^m = 2$ $\therefore m\left(\frac{x}{a}\right)^{m-1} \cdot \frac{1}{a} + m\left(\frac{y}{b}\right)^{m-1} \cdot \frac{1}{b} \cdot \frac{dy}{dx} = 0$ $\therefore \text{at } (a, b),$ $m\left(\frac{a}{a}\right)^{m-1} \cdot \frac{1}{a} + m\left(\frac{b}{b}\right)^{m-1} \cdot \frac{1}{b} \cdot \frac{dy}{dx} = 0$ $\therefore m \cdot \frac{1}{a} + m \cdot \frac{1}{b} \cdot \frac{dy}{dx} = 0$ $\therefore \text{slope of tangent at } (a, b) = \frac{dy}{dx} = -\frac{b}{a}$ $\therefore \text{the equation of tangent is}$ $y - b = -\frac{b}{a}(x - a)$ $\therefore ay - ab = -bx + ab$ $\therefore bx + ay = 2ab$ $\therefore \frac{x}{a} + \frac{y}{b} = 2$	<p>1</p> <p>1/2</p> <p>1/2</p> <p>1</p> <p>1/2</p> <p>1/2</p>	4



Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
6)	e)	Divide 80 into two parts such that their product is maximum.		
	Ans.	<p>Let x, y be the numbers. But <math>x + y = 80</math> i. e., <math>y = 80 - x</math> To maximize, <math>p = xy = x(80 - x)</math> <math>\therefore p = 80x - x^2</math> <math>\therefore \frac{dp}{dx} = 80 - 2x</math> <math>\therefore \frac{d^2p}{dx^2} = -2</math></p> <p>For stationary values, <math>\frac{dp}{dx} = 0</math> <math>\therefore 80 - 2x = 0</math> or <math>80 = 2x</math> <math>\therefore x = 40</math></p> <p>At <math>x = 40</math>, <math>\frac{d^2p}{dx^2} = -2 &lt; 0</math> <math>\therefore</math> At <math>x = 40</math>, <math>p</math> has maximum value.</p> <p>-----</p>	<p>1 1/2 1/2 1/2 1/2 1/2</p>	4
f)	Two points A(1, 4) and B(9, 12) are on the parabola $y^2 = 16x$ . Show that the area enclosed between the chord AB and the parabola is $\frac{16}{3}$ .			
	Ans.	<p><math>\frac{x-1}{9-1} = \frac{y-4}{12-4}</math> <math>\therefore x-1 = y-4</math> <math>\therefore y = x+3</math></p> <p><b>Note: Students may use another form to find the equation of this line, such as slope point form.</b></p> <p><math>\therefore A = \int_a^b (y_2 - y_1) dx</math> <math>= \int_1^9 (4\sqrt{x} - x - 3) dx</math> <math>= \left[ 4 \cdot \frac{2}{3} x^{3/2} - \frac{x^2}{2} - 3x \right]_1^9</math> <math>= \left[ \frac{8}{3} (9)^{3/2} - \frac{9^2}{2} - 3(9) \right] - \left[ \frac{8}{3} - \frac{1}{2} - 3 \right]</math> <math>= \frac{16}{3}</math> or 5.333</p>	<p>1 1 1/2 1/2</p>	4



Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
6)		<p style="text-align: center;"><b>OR</b></p> $\frac{x-1}{9-1} = \frac{y-4}{12-4}$ $\therefore x-1 = y-4$ $\therefore y = x+3$ $\therefore A = \int_a^b (y_2 - y_1) dx$ $= \int_1^9 (x+3-4\sqrt{x}) dx$ $= \left[ \frac{x^2}{2} + 3x - 4 \cdot \frac{2}{3} x^{3/2} \right]_1^9$ $= \left[ \frac{9^2}{2} + 3(9) - \frac{8}{3}(9)^{3/2} \right] - \left[ \frac{1}{2} + 3 - \frac{8}{3} \right]$ $= -\frac{16}{3} \quad \text{or} \quad -5.333$ $\therefore \text{Area } A = \frac{16}{3} \quad \text{or} \quad 5.333$ <hr style="border-top: 1px dashed black;"/> <hr style="border-top: 1px dashed black;"/> <hr style="border-top: 1px dashed black;"/> <hr style="border-top: 1px dashed black;"/> <p style="text-align: center;"><b>Important Note</b></p> <p>In the solution of the question paper, wherever possible all the possible alternative methods of solution are given for the sake of convenience. Still student may follow a method other than the given herein. <b>In such case, FIRST SEE whether the method falls within the scope of the curriculum</b>, and THEN ONLY give appropriate marks in accordance with the scheme of marking.</p> <hr style="border-top: 1px dashed black;"/> <hr style="border-top: 1px dashed black;"/>	<p style="text-align: center;">1</p> <p style="text-align: center;">1</p> <p style="text-align: center;">1</p> <p style="text-align: center;">1/2</p> <p style="text-align: center;">1/2</p>	<b>4</b>



Area Under Standard Normal Curve

	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0	0	0.004	0.008	0.012	0.016	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.091	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.148	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.17	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.195	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.219	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.258	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.291	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.334	0.3365	0.3389
1	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.377	0.379	0.381	0.383
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.398	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.437	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.475	0.4756	0.4761	0.4767
2	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.483	0.4834	0.4838	0.4842	0.4846	0.485	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.489
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.492	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.494	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.496	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.497	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.498	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.499	0.499

e. g., The value of  $P(0 \leq z \leq 0.45)$  is 0.1736.