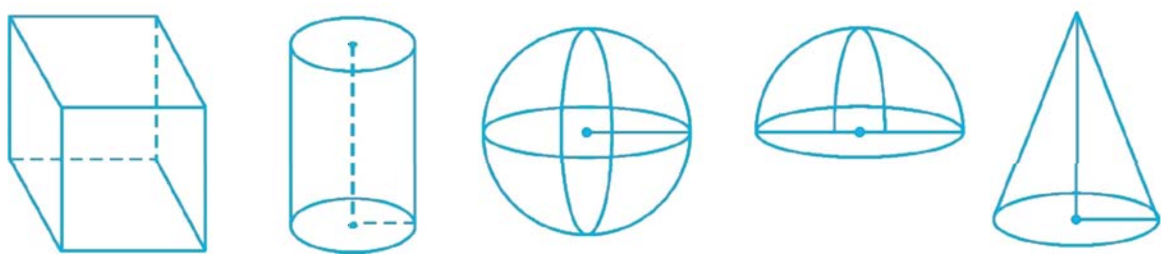
SURFACE AREA AND VOLUME 14

If there is a problem you can't solve, then there is an easier problem you can solve. Find it.

- George Polya

14.1 Introduction

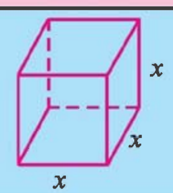
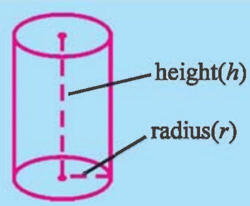
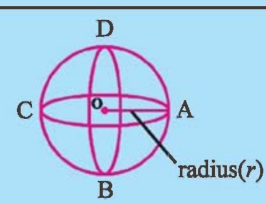
We are already familiar with the surface area and volume of some regular solids like cuboid, cylinder, sphere, hemisphere and right circular cone (see figure 14.1)

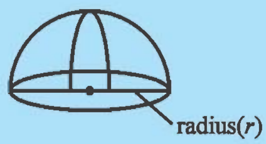
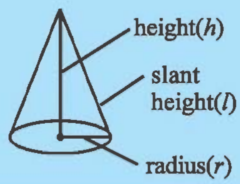


(i) (ii) (iii) (iv) (v)

Figure 14.1

Surface area of some familiar solids :

Sr. No.	Solid	Figure	State of solid	Surface area
1.	Cube		Open cube Closed cube	$5x^2$ $6x^2$
2.	Cylinder		Curved surface area Total surface area	$2\pi rh$ $2\pi r(r + h)$
3.	Sphere		Surface area	$4\pi r^2$

Sr. No.	Solid	Figure	State of solid	Surface area
4.	Hemisphere		Open hemisphere Closed hemisphere	$2\pi r^2$ $3\pi r^2$
5.	Right circular cone		Lateral surface area Total surface area	πrl $\pi r(r + l)$

Note : For simple calculations take $\pi = \frac{22}{7}$ unless otherwise stated.

In our daily life, we come across some solids made up of combinations of two or more of the basic solids as shown in figure 14.1.

We have seen the container on the back of truck or on the train which contains either water or oil or milk. The shape of the container is made of a cylinder with two hemispheres at its ends.

In our science laboratory we have seen a test-tube. This tube is also a combination of a cylinder and a hemisphere at one end.

We have seen a toy top also, it is a combination of a cone and hemisphere at the base of cone.

14.2 Surface Area of a Combination of Solids

Let us consider the cylindrical vessel (see figure 14.2). How do we find the surface area of such a solid ? Whenever we come across a new problem, we first divide (or break it down) into smaller problems which we have solved earlier. We can see that this solid is made up of a cylinder with a conical lid surmounted on it. It looks like what we have in figure 14.3 after we put both the pieces together. To find the surface area of cylindrical vessel, we have to find the surface area of a cone and the surface area of a cylinder individually.

Figure 14.2

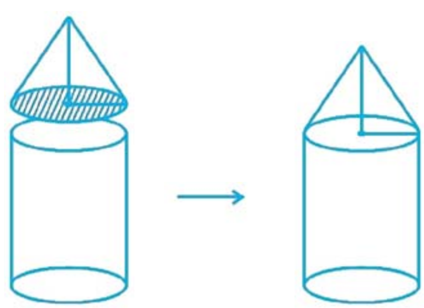


Figure 14.3

Total surface area of a cylindrical vessel (TSA) = Curved surface area of cylinder (CSA) + Curved surface area of cone (CSA).

Let us consider another solid. Suppose we are making a toy by putting cone and hemisphere together. Now let us see how to find the total surface area of this toy. (See figure 14.4)

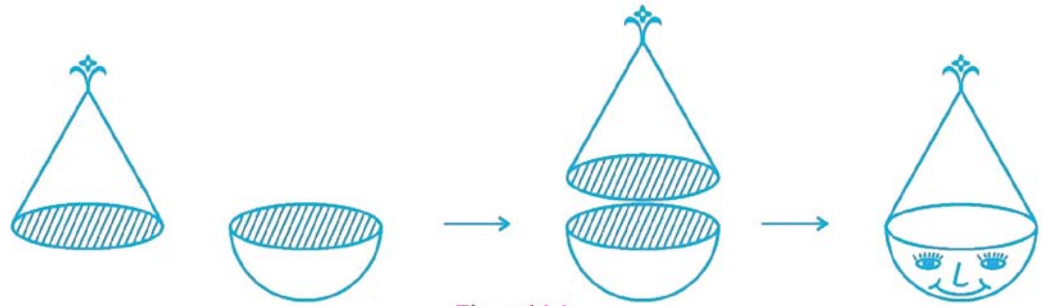


Figure 14.4

First we take a cone and a hemisphere and bring their flat faces together, of course we take the radius of the base of the cone and radius of the hemisphere same. So the steps are as shown in figure 14.4. At the end we get a nice round-bottomed toy. Now if we want to find the surface area of this toy, what should we do ? We need to find total surface area of the toy. We need the curved surface area of a cone and curved surface area of a hemisphere. So we get,

Total surface area of the toy = Curved surface area of cone + CSA of hemisphere.

Now let us learn some examples.

Example 1 : How many square meters of cloth is required to prepare four conical tents of diameter 8 *m* and height 3 *m*. ($\pi = 3.14$)

Solution : Here diameter of the tent is 8 *m*, so the radius is 4 *m* and height of the tent is 3 *m*.

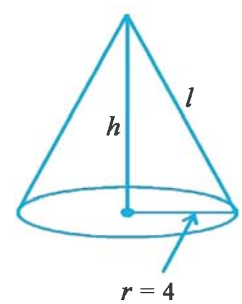


Figure 14.5

Now the slant height $l = \sqrt{h^2 + r^2}$

$$\therefore \quad l = \sqrt{9 + 16} = 5\ m$$

Now, the curved surface area of cone $= \pi rl$

$$= 3.14 \times 4 \times 5$$

$$= 62.8\ m^2$$

So, the cloth required for one tent 62.8 m^2.

Therefore the cloth required for four tents is 251.2 m^2.

Example 2 : A cylinder has hemispherical ends having radius 14 *cm* and height 50 *cm*. Find the total surface area.

Solution : Here the radius of cylinder and hemisphere is 14 *cm* and the height is 50 *cm* as in figure 14.6.

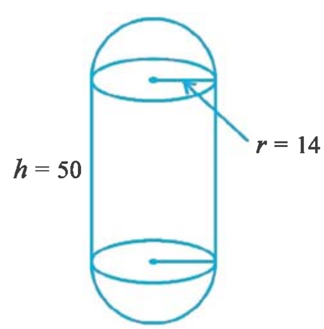


Figure 14.6

Total surface area of the solid composed of a cylinder and two hemispherical ends,

$$= \text{CSA of cylinder} + 2 \times \text{CSA of hemispheres.}$$

$$= 2\pi rh + 2(2\pi r^2)$$

$$= 2\pi r\,(h + 2r)$$

$$= 2 \times \frac{22}{7} \times 14\ (50 + 28)$$

$$= 6864\ cm^2$$

Example 3 : A box is made up of a cylinder surmounted by a cone. The radius of the cylinder and cone is 12 *cm* and slant height of the cone is 13 *cm*. The height of the cylinder is 11 *cm*. Find the curved surface area of the box.

Solution : Here radius of the cylinder = radius of the cone = 12 *cm* = r

Height of the cylinder = 11 *cm*. Slant height of the cone (l) = 13 *cm*.

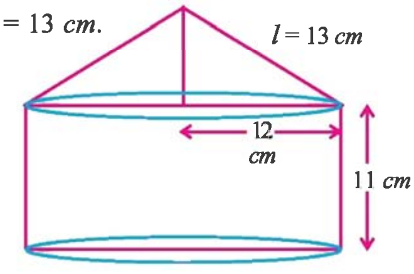


Figure 14.7

∴ Total curved surface area of the given solid

= CSA of the cylinder + CSA of the cone

$= 2\pi rh + \pi rl$

$= \pi r(2h + l)$

$= \frac{22}{7} \times 12(2 \times 11 + 13) = 1320\ cm^2$

Thus, the curved surface area of the given solid is 1320 cm^2.

Example 4 : A metallic cylinder has diameter 1 *m* and height 3.2 *m*. Find the cost of painting its outer surface at the rate of ₹ 35 per square meter. ($\pi = 3.14$)

Solution : The diameter of the cylinder is 1 *m*.

∴ The radius (r) of the cylinder = 0.5 *m* and the height (h) of the cylinder is 3.2 *m*.

∴ The total surface area of the cylinder (including top and bottom)

$= 2\pi r(r + h)$

$= 2 \times 3.14 \times 0.5\ (0.5 + 3.2) = 11.618\ m^2$

Now, the cost of painting is ₹ 35 per square meter, so the total cost of painting this cylinder

= ₹ 35 × 11.618 = ₹ 406.63

∴ The total cost of painting this cylinder is ₹ 407 (to nearest rupee).

Example 5 : The total surface area of a hemisphere is 763.72 cm^2. Find its diameter.

Solution : Let r be the radius of the hemisphere.

∴ Total surface area of the hemisphere $= 3\pi r^2$

∴ $r^2 = \frac{763.72}{3\pi} = \frac{763.72}{3} \times \frac{7}{22} = 81.033 = 81$ (approx.)

∴ $r = 9\ cm$

∴ The diameter $= 2r = 18\ cm$

∴ The diameter of the hemisphere is 18 *cm*.

Example 6 : The radius of a conical shaped dome of a temple is 7 *m* and its height is 24 *m*. Find the cost of painting both the sides (inside and outside) of the dome of the temple at the rate of ₹ 15 per square meter. (neglect thickness)

Solution : Here the height of the dome is 24 *m* and the radius of the dome is 7 *m*.

∴ The slant height is $l = \sqrt{24^2 + 7^2} = 25\ m$

The curved surface area of cone $= \pi rl$

$= \frac{22}{7} \times 7 \times 25$

$= 550\ m^2$

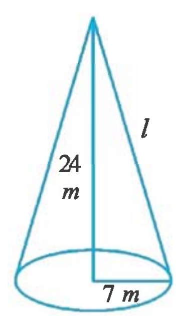


Figure 14.8

Now, the cost of painting is ₹ 15 per square meter.

∴ The cost of painting the outer side of the dome is 15 × 550 = ₹ 8250

The total cost of painting both the sides is 2 × 8250 = ₹ 16500

Example 7 : The total surface area of a solid composed of a cone with hemispherical base is 361.1 cm^2. (π = 3.14) The dimension are shown in figure 14.9. Find the total height of the solid.

Solution : Suppose the radius of the hemisphere and the base of cone is r.

$\therefore$ The total surface area of the given solid = $\pi rl + 2\pi r^2$

$\therefore$ $361.1 = 3.14(r \times 13 + 2 \cdot r^2)$

$\therefore$ $\frac{361.1}{3.14} = 13r + 2r^2$

$\therefore$ $115 = 2r^2 + 13r$

$\therefore$ $2r^2 + 13r - 115 = 0$

$\therefore$ $(r - 5)(2r + 23) = 0$

$\therefore$ $r = 5$ or $r = \frac{-23}{2}$

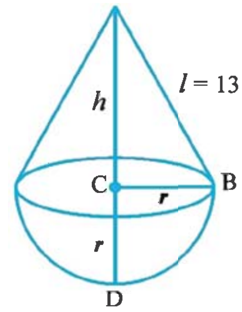


Figure 14.9

But radius is positive. So, $r = 5$ cm

Now, from the figure 14.9 the height of the cone = $h = \sqrt{l^2 - r^2}$

$= \sqrt{169 - 25}$

$= 12$ cm

$\therefore$ The total height of the solid = $h + r = 17$ cm

EXERCISE 14.1

1. A toy is made by mounting a cone onto a hemisphere. The radius of the cone and a hemisphere is 5 cm. The total height of the toy is 17 cm. Find the total surface area of the toy.

2. A show-piece shown in figure 14.10 is made of two solids - a cube and a hemisphere. The base of the block is a cube with edge 7 cm and the hemisphere fixed on the top has diameter 5.2 cm. Find the total surface area of the piece.

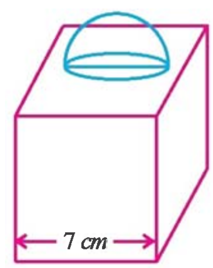


Figure 14.10

3. A vessel is in the form of a hemisphere mounted on a hollow cylinder. The diameter of the hemisphere is 21 cm and the height of vessel is 25 cm. If the vessel is to be painted at the rate of ₹ 3.5 per cm^2, then find the total cost to paint the vessel from outside.

4. Chirag made a bird-bath for his garden in the shape of a cylinder with a hemispherical depression at one end, (see the figure 14.11). The height of the cylinder is 1.5 m and its radius is 50 cm. Find the total area of the bird-bath. (π = 3.14)

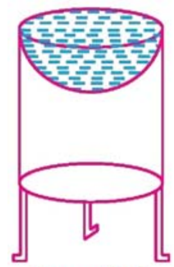


Figure 14.11

5. A solid is composed of a cylinder with hemisperical ends on both the sides. The radius and the height of the cylinder are 20 cm and 35 cm respectively. Find the total surface area of the solid.

6. The radius of a conical tent is 4 m and slant height is 5 m. How many meters of canvas of width 125 cm will be used to prepare 12 tents ? If the cost of canvas is ₹ 20 per meter, then what is total cost of 12 tents ? (π = 3.14)

7. If the radius of a cone is 60 cm and its curved surface area is 23.55 m^2, then find its slant height. (π = 3.14)

8. The cost of painting the surface of sphere is ₹ 1526 at the rate of ₹ 6 per m^2. Find the radius of sphere.

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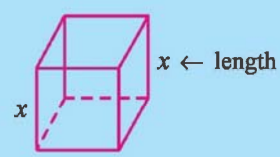
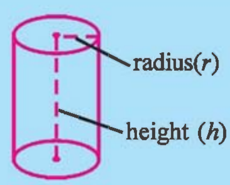
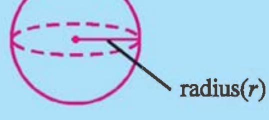
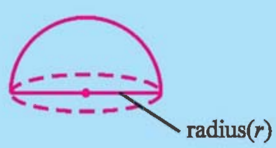
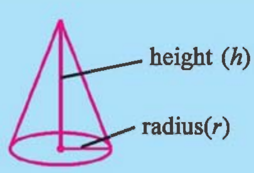
14.3 Volume of a combination of Solids

We have seen how to find the surface area of given solids made up of a combination of two basic solids in the previous section. Here we will study how to find the volume of such solids. It is noted that in the calculation of surface area, we cannot add the surface areas of the two constituents, because some part of the surface area have disappeared in the process of joining them. But this will not happen in the calculation of the volume. The volume of the solid formed by joining two basic solids will actually be the sum of the volumes of the constituents.

Note that 1 litre = 1000 cm^3. 1 m^3 = 1000 litre

Volume of some familiar solids

Sr. No.	Solid	Figure	Volume
1.	Cube		x^3
2.	Cylinder		$\pi r^2 h$
3.	Sphere		$\frac{4}{3}\pi r^3$
4.	Hemisphere		$\frac{2}{3}\pi r^3$
5.	Cone		$\frac{1}{3}\pi r^2 h$

Let us see some examples to understand the concept given above.

Example 8 : What will be the volume of the cone whose height is 21 *cm* and radius of the base is 6 *cm* ?

Solution : Volume of cone $= \frac{1}{3}\pi r^2 h$

$$= \frac{1}{3} \times \frac{22}{7} \times 6 \times 6 \times 21$$

$$= 792\ cm^3$$

$\therefore$ The volume of the cone is 792 cm^3.

Example 9 : Find the capacity of a cylindrical water tank whose radius is 2.1 *m* and height 5 *m*.

Solution : Here radius of the cylinder is $r = 2.1\ m$ and height $h = 5\ m$.

Volume of the cylindrical tank $= \pi r^2 h$

$$= \frac{22}{7} \times 2.1 \times 2.1 \times 5 = 69.3\ m^3$$

Now, 1 m^3 = 1000 litres

$\therefore$ 69.3 m^3 = 69.3 × 1000 = 69300 litres

Example 10 : How many maximum litres of petrol can be contained in a cylindrical tank with hemispherical ends having radius 0.42 *m* and total height 3.84 *m* ?

Solution : Here radius of the hemisphere $r = 0.42\ m$ = radius of the cylinder.

Now, the height of cylinder = Total height – 2(radius of hemisphere)

$$= 3.84 - 2(0.42) = 3\ m.$$

The volume of the cylindrical tank with hemispherical ends

$$= \pi r^2 h + \frac{4}{3}\pi r^3 \qquad \left(2 \times \frac{2}{3}\pi r^3\right)$$

$$= \frac{22}{7} \times (0.42)^2 \times 3 + \frac{4}{3} \times \frac{22}{7} \times (0.42)^3$$

$$= 22 \times 0.06 \times 0.42 \times 3 + 4 \times 22 \times 0.02 \times 0.42 \times 0.42$$

$$= 1.6632 + 0.310464 = 1.973664\ m^3.$$

Now, 1 m^3 = 1000 litres

$\therefore$ 1.973664 m^3 = 1.973664 × 1000

= 1973.66 litres

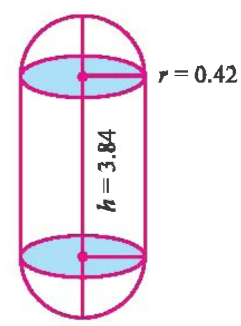


Figure 14.12

Example 11 : A common plot of a society is in the form of circle having diameter 5.6 *m*. Find how many cubic meters of soil is required to raise the level of ground by 25 *cm* ?

Solution : Here the diameter of the circle is 5.6 *m*.

$\therefore$ The radius r is 2.8 *m*

The area of a circle is πr^2, and we want to raise this region by 25 *cm* i.e. 0.25 *m* (say)

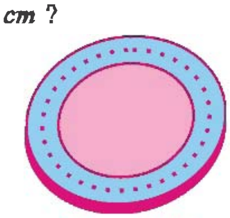


Figure 14.13

$\therefore$ Volume of the soil required to raise the ground $= \pi r^2 h$

$$= \frac{22}{7} \times 2.8 \times 2.8 \times 0.25$$

$$= 6.16\ m^3$$

$\therefore$ The volume of the soil required is 6.16 m^3.

Example 12 : The volume of a cone is 9504 cm^3 and the radius of the base is 18 *cm*. Find the height of cone.

Solution : Here the radius of base of cone is 18 *cm* and the volume of the cone is 9504 cm^3.

The volume of the cone = $\frac{1}{3}\pi r^2 h$

$\therefore \quad 9504 = \frac{1}{3} \times \frac{22}{7} \times 18 \times 18 \times h$

$\therefore \quad h = \frac{9504 \times 3 \times 7}{22 \times 18 \times 18}$

$\therefore \quad h = 28\ cm$

$\therefore$ The height of the cone is 28 *cm*.

Example 13 : Mayank, an engineering student, was asked to make a model shaped like a cylinder with two cones attached at both ends with thin film-sheet. The radius of the model is 4 *cm* and the total height is 13 *cm*. If each cone has height 3 *cm*, find the volume of the air contained in the model.

Solution : Here radius of the cone and cylinder is $r = 4\ cm$ and the height of cone is $h = 3\ cm$. The height of the cylinder $H = 13 - 2(3) = 7\ cm$

As shown in the figure 14.14, the total volume is divided into three parts, namely two conical parts and one cylindrical part.

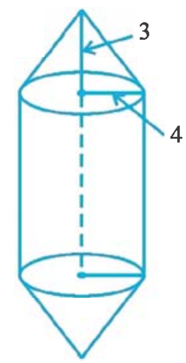


Figure 14.14

The volume of the cone $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 4 \times 4 \times 3$

$= 50.29\ cm^3$

The volume of the cylinder $= \pi r^2 H$

$= \frac{22}{7} \times 4 \times 4 \times 7$

$= 352\ cm^3$

$\therefore$ The total volume of the model = 2 × volume of cone + volume of cylinder

$= 2 \times 50.29 + 352$

$= 452.58\ cm^3$.

The volume of air in the model is $452.58\ cm^3$.

EXERCISE 14.2

1. The curved surface area of a cone is $550\ cm^2$. If its diameter is 14 *cm*, find its volume.
2. A solid is in the form of cone with hemispherical base. The radius of the cone is 15 *cm* and the total height of the solid is 55 *cm*. Find the volume of the solid. ($\pi = 3.14$)
3. How many litres of milk can be stored in a cylindrical tank with radius 1.4 *m* and height 3 *m* ?
4. The spherical balloon with radius 21 *cm* is filled with air. Find the volume of air contained in it.
5. A solid has hemi-spherical base with diameter 8.5 *cm* and it is surmounted by a cylinder with height 8 *cm* and diameter of cylinder is 2 *cm*. Find the volume of this solid. ($\pi = 3.14$)
6. A playing top is made up of steel. The top is shaped like a cone surmounted by a hemisphere. The total height of top is 5 *cm* and the diameter of the top is 3.5 *cm*. Find the volume of the top.
7. How many litres of petrol will be contained in a closed cylindrical tank with hemisphere at one end having radius 4.2 *cm* and total height 27.5 *cm* ?

8. The capacity of a cylindrical tank at a petrol pump is 57750 litres. If its diameter is 3.5 *m*, find the height of cylinder.
9. A hemispherical pond is filled with 523.908 m^3 of water. Find the maximum depth of pond.
10. A gulab-jamun contain 40 % sugar syrup in it. Find how much syrup would be there in 50 gulab-jamuns, each shaped like a cylinder with two hemispherical ends with total length 5 *cm* and diameter 2.8 *cm*.
11. The height and the slant height of a cone are 12 *cm* and 20 *cm* respectively. Find its volume. ($\pi = 3.14$)
12. Find the total volume of a cone having a hemispherical base. If the radius of the base is 21 *cm* and height 60 *cm*.
13. If the slant height of a cone is 18.7 *cm* and the curved surface area is 602.8 cm^2, find the volume of cone. ($\pi = 3.14$)
14. If the surface area of a spherical ball is 1256 cm^2, then find the volume of sphere. (Take $\pi = 3.14$)

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14.4 Conversion of a Solid from one Shape to Another

We know that some of the solids can be melted and can be converted into another shapes, for example wax candle, iron piece, copper etc.

Let us understand the concept of conversion of a solid form into another solid form by examples.

Example 14 : How many balls of radius 0.5 *cm* can be prepared by melting a metal cylinder of radius 5 *cm* and height 7 *cm* ?

Solution : Radius (r) of the cylinder is 5 *cm* and the height (h) is 7 *cm*.

$\therefore$ The volume of the cylinder $= \pi r^2 h$

$$= \frac{22}{7} \times 5 \times 5 \times 7 = 550\ cm^3$$

Let the radius of a ball be R.

Now, the volume of a ball $= \frac{4}{3}\pi R^3$

$$= \frac{4}{3} \times \frac{22}{7} \times (0.5)^3 = 0.5238\ cm^3$$

Now, the volume of 1 ball $= 0.5238\ cm^3$

$\therefore$ The number of balls $= \dfrac{\text{Volume of the cylinder}}{\text{Volume of a ball}}$

$$= \frac{550}{0.5238} = 1050 \text{ (approximately)}$$

$\therefore$ Total number of balls formed is 1050.

Example 15 : A metallic sphere of radius 3.6 *cm* is melted and a wire of diameter 0.4 *cm* of uniform cross-section is drawn from it. Find the length of the wire.

Solution : Let the length (or height) of the wire be h and the radius be r. Also the radius of the sphere is R.

$\therefore$ $r = 0.2\ cm$, $R = 3.6\ cm$

$\therefore$ The volume of the wire = The volume of the sphere

$\therefore$ $\pi r^2 h = \frac{4}{3}\pi R^3$

$\therefore$ $h = \frac{4}{3} \times \frac{3.6 \times 3.6 \times 3.6}{0.2 \times 0.2}$

$= 1555.2\ cm$

$= 15.552\ m$

$\therefore$ The length of the wire is 15.552 *m*.

Example 16 : A hemispherical tank full of water is emptied by a pipe at the rate of $14\frac{2}{7}$ litres per second. How much time will it take to empty three fourth of the tank, if it is 4 *m* in diameter ?

Solution : Radius of hemisphere = 2 *m*

The volume of the tank $= \frac{2}{3}\pi r^3$

$= \frac{2}{3} \times \frac{22}{7} \times (2)^3$

$= \frac{352}{21}\ m^3$

So, the volume of the water to be emptied $= \frac{3}{4} \times \frac{352}{21} \times 1000 = \frac{88}{7} \times 1000$

$= \frac{88000}{7}$ litres

Since $\frac{100}{7}$ litres of water is emptied in 1 second.

$\therefore$ $\frac{88000}{7}$ litres of water will be emptied in $\frac{88000}{7} \times \frac{7}{100} = 880$ seconds

Example 17 : A 30 *m* deep cylindrical well with diameter 7 *m* is dug and the soil obtained by digging is evenly spread out to form a platform 30 *m* × 10 *m*. Find the height of the platform.

Solution : The radius of the well is $r = \frac{7}{2} = 3.5\ m$

The volume of the soil digged out from the well $= \pi r^2 h$ **(h = height of the well)**

$= \frac{22}{7} \times 3.5 \times 3.5 \times 30$

$= 1155\ m^3$

The volume of the soil = The volume of the platform

$\therefore$ $1155 = l \times b \times H = 30 \times 10 \times H$ **(H = height of the platform)**

$\therefore$ $H = \frac{1155}{30 \times 10} = 3.85\ m$

$\therefore$ The height of the platform is 3.85 *m*

Example 18 : How many spherical balls of diameter 0.5 *cm* can be cast by melting a metal cone with radius 6 *cm* and height 14 *cm* ?

Solution : Radius of cone = 6 *cm* = R

The height of the cone = h = 14 *cm*, the radius of the sphere = r = 0.5 *cm* = $\frac{1}{2}$ *cm*

Now, Number of balls $= \frac{\text{The volume of the cone}}{\text{The volume of the sphere}}$

$= \frac{\frac{1}{3}\pi R^2 h}{\frac{4}{3}\pi r^3} = \frac{(6)^2 \times 14}{4 \times \left(\frac{1}{2}\right)^3}$

$= 1008$

$\therefore$ The number of spherical balls is 1008.

EXERCISE 14.3

1. A hemispherical bowl of internal radius 12 *cm* contains some liquid. This liquid is to be filled into cylindrical bottles of diameter 4 *cm* and height 6 *cm*. How many bottles can be filled with this liquid ?
2. A cylindrical container having diameter 16 *cm* and height 40 *cm* is full of ice-cream. The ice-cream is to be filled into cones of height 12 *cm* and diameter 4 *cm*, having a hemispherical shape on the top. Find the number of such cones which can be filled with the ice-cream.
3. A cylindrical tank of diameter 3 *m* and height 7 *m* is completely filled with groundnut oil. It is to be emptied in 15 tins each of capacity 15 litres. Find the number of such tins required.
4. A cylinder of radius 2 *cm* and height 10 *cm* is melted into small spherical balls of diameter 1 *cm*. Find the number of such balls.
5. A metallic sphere of radius 15 *cm* is melted and a wire of diameter 1 *cm* is drawn from it. Find the length of the wire.
6. There are 45 conical heaps of wheat, each of them having diameter 80 *cm* and height 30 *cm*. To store the wheat in a cylindrical container of the same radius, what will be the height of cylinder ?
7. A cylindrical bucket, 44 *cm* high and having radius of base 21 *cm*, is filled with sand. This bucket is emptied on the ground and a conical heap of sand is formed. If the height of the conical heap is 33 *cm*, find the radius and the slant height of the heap.

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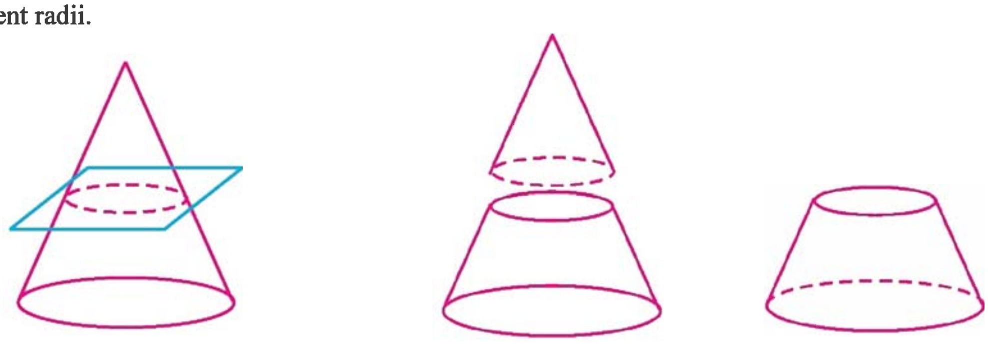
14.5 Frustum of a Cone

In section 14.2, we had seen the objects that are formed when two basic solids were joined together. Here we will do something different. We will take a right circular cone and remove a portion of it. We can do this in many ways. But we will take one particular case that we will remove a smaller right circular cone by cutting the given cone by a plane parallel to its base. We have seen the glasses or tumblers, in general, used for drinking water, are of this shape. (See figure 14.13)

Figure 14.13

Activity : Take some clay or paper or any other such material (like plastic) and form a cone. Cut it with knife parallel to its base. Remove the smaller cone. What will be the remaining portion left ? The portion left with solid is called a **frustum** of the cone. It has two circular ends with different radii.



A cone sliced by a plane parallel to base　　The two part separated　　Frustum of a cone

Figure 14.14

So, given a cone, when we slice (or cut) through it with a plane parallel to its base and remove the cone that is formed on one side of the plane, the part which is left on the other side of the plane is called a **frustum of the cone.** (Frustum is a latin word which means "piece cut off" and its plural is 'frusta'.) (See figure 14.14)

Now let us see how to find the surface area and volume of a frustum of a cone by an example given below :

Example 19 : The radii of the ends of a frustum are 32 *cm* and 8 *cm* and the height of the frustum of the cone is 54 *cm*. Find its volume, the curved surface area and the total surface area.

(See figure 14.15)

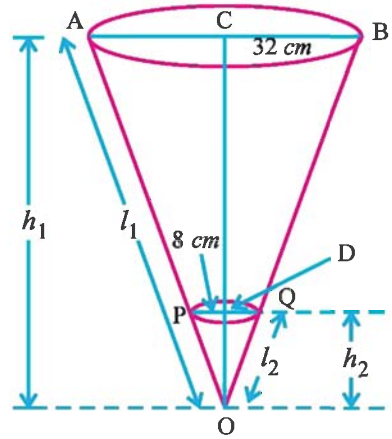


Figure 14.15

Solution : We can see that the volume of the frustum is the difference of volumes of two right circular cones OAB and OPQ. Let the heights and the slant heights of cones OAB and OPQ be respectively h_1, h_2 and l_1, l_2 and radii r_1 and r_2.

Here we have $r_1 = 32$ *cm*, $r_2 = 8$ *cm*

The height of the frustum is 54 *cm*.

Also, $h_1 = 54 + h_2$ **(i)**

Now, ΔOCB and ΔODQ are similar right angle triangles.

$\therefore \quad \frac{h_1}{h_2} = \frac{r_1}{r_2} = \frac{32}{8} = 4$

$\therefore \quad h_1 = 4h_2$

$\therefore \quad 4h_2 = 54 + h_2$. So $h_2 = 18$ **(by (i))**

Now, the volume of the frustum = Volume of the cone OAB − Volume of the cone OPQ

$= \frac{1}{3}\pi r_1^2 h_1 - \frac{1}{3}\pi r_2^2 h_2$

$= \frac{1}{3}\pi \times (4r_2)^2 \times 4h_2 - \frac{1}{3}\pi \times r_2^2 \times h_2$

$= \frac{1}{3} \times \pi \times 63r_2^2 \times h_2$

$= \frac{1}{3} \times \frac{22}{7} \times 63 \times 8 \times 8 \times 18 = 76032\ cm^3$

Now, slant heights l_1 and l_2 of cones OAB and OPQ respectively are given by

$l_2 = \sqrt{8^2 + 18^2} = \sqrt{388} = 19.698$ *cm* (approximately)

$l_1 = \sqrt{32^2 + 72^2} = \sqrt{6208} = 78.79$ *cm* (approximately)

Therefore the curved surface area of the frustum $= \pi r_1 l_1 - \pi r_2 l_2$

$= \pi \times 4r_2 \times 4l_2 - \pi \times r_2 \times l_2$

$= \frac{22}{7} \times 15 \times 8 \times 19.698$

$= 7428.96\ cm^2$

The total surface area of the frustum = the curved surface area $+ \pi r_1^2 + \pi r_2^2$

$= 7428.96 + \frac{22}{7} \times (32)^2 + \frac{22}{7} \times (8)^2$

$= 7428.96 + 3218.29 + 201.14$

$= 10848.39\ cm^2$

Note : Let h be the height, l be the slant height, r_1 and r_2 be radii of the ends ($r_1 > r_2$) of a frustum of a cone. Then we will accept the following formulae for the volume of frustum, the curved surface area and total surface area of the frustum as given below :

(i) Volume of the frustum of a cone = $\frac{1}{3}\pi h\ [r_1^2 + r_2^2 + r_1 r_2]$

(ii) The curved surface area of a frustum of a cone = $\pi(r_1 + r_2) \cdot l$ **where** $l = \sqrt{h^2 + (r_1 - r_2)^2}$

(iii) Total surface area of a frustum of a cone = $\pi l(r_1 + r_2) + \pi r_1^2 + \pi r_2^2$,

where $l = \sqrt{h^2 + (r_1 - r_2)^2}$

Let us solve example 19 by the above formula :

$$\text{Volume of the frustum} = \frac{1}{3}\pi h\ [r_1^2 + r_2^2 + r_1 r_2]$$

$$= \frac{1}{3} \times \frac{22}{7} \times 54\ [(32)^2 + (8)^2 + 32 \times 8]$$

$$= \frac{1}{3} \times \frac{22}{7} \times 54 \times 1344 = 76032\ cm^3$$

The curved surface area of frustum $= \pi(r_1 + r_2) \cdot l$ where

$$l = \sqrt{h^2 + (r_1 - r_2)^2} = \sqrt{54^2 + (32 - 8)^2} = 59.09\ cm \text{ (approximately)}$$

$\therefore$ Curved surface area of the frustum $= \frac{22}{7}(32 + 8) \times (59.09) = 7428.46\ cm^2$

The total surface area of the frustum $=$ curved surface area $+ \pi r_1^2 + \pi r_2^2$

$$= 7428.46 + \frac{22}{7} \times (32)^2 + \frac{22}{7} \times (8)^2$$

$$= 7428.46 + 3218.29 + 201.14 = 10847.89\ cm^2$$

Example 20 : A drinking glass is in the shape of frustum of a cone of height 21 cm. The radii of its two circular ends are 3 cm and 2 cm. Find the capacity of the glass.

Solution : The height of the glass is 21 cm and the radii of two circular ends are 3 cm and 2 cm.

$\therefore$ $h = 21\ cm$, $r_1 = 3\ cm$ and $r_2 = 2\ cm$

The capacity of the glass = the volume of the glass $= \frac{1}{3}\pi h\ [r_1^2 + r_2^2 + r_1 r_2]$

$$= \frac{1}{3} \times \frac{22}{7} \times 21 \times [3^2 + 2^2 + 3 \times 2]$$

$$= \frac{1}{3} \times \frac{22}{7} \times [19] \times 21$$

$$= 418\ cm^3$$

Example 21 : An oil funnel made of tin sheet consists of a 20 cm long cylindrical portion attached to a frustum of a cone. If the total height is 40 cm, diameter of the cylindrical portion is 14 cm and the diameter of the top of the funnel is 24 cm, find the area of the tin sheet required to make the funnel. (See fig. 14.16)

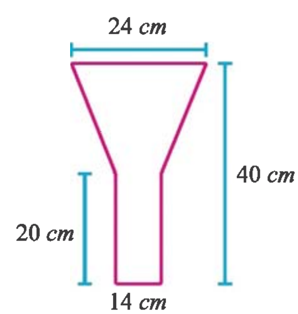


Figure 14.16

Solution : The curved surface area of the cylinder $= 2\pi rh$

$$= 2 \times \frac{22}{7} \times 7 \times 20$$

$$= 880\ cm^2$$

The curved surface area of the frustum $= \pi(r_1 + r_2) \cdot l$

Here $l = \sqrt{h^2 + (r_1 - r_2)^2}$, $r_1 = 12\ cm$, $r_2 = 7\ cm$ and $h = 20\ cm$

$\therefore \quad l = \sqrt{400 + (12 - 7)^2} = \sqrt{425} = 5\sqrt{17}$

$\therefore$ The curved surface area of the frustum $= \pi(r_1 + r_2) \cdot l$

$$= \frac{22}{7} \times (12 + 7) \times 5\sqrt{17}$$

$$= 1231.04\ cm^2$$

$\therefore$ The total area of the tin sheet $= 880 + 1231.04 = 2111.04\ cm^2$

EXERCISE 14.4

1. A metal bucket is in the shape of a frustum of a cone, mounted on a hollow cylindrical base made of the same metalic sheet. The total vertical height of the bucket is 40 *cm* and that of cylindrical base is 10 *cm*, radii of two circular ends are 60 *cm* and 20 *cm*. Find the area of the metalic sheet used. Also find the volume of water the bucket can hold. ($\pi = 3.14$)
2. A container, open from the top and made up of a metal sheet is the form of frustum of a cone of height 30 cm with radii 30 *cm* and 10 *cm*. Find the cost of the milk which can completely fill container at the rate of ₹ 30 per litre. Also find the cost of metal sheet used to make the container, if it costs ₹ 50 per 100 cm^2. ($\pi = 3.14$)

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EXERCISE 14

1. A tent is in the shape of cylinder surmounted by a conical top. If the height and the radius of the cylindrical part are 3.5 *m* and 2 *m* respectively and the slant height of the top is 3.5 *m*, find the area of the canvas used for making the tent. Also find the cost of canvas of the tent at the rate of ₹ 1000 per m^2.
2. A metallic sphere of radius 5.6 *cm* is melted and recast into the shape of a cylinder of radius 6 *cm*. Find the height of the cylinder.
3. How many spherical balls of radius 2 *cm* can be made out of a solid cube of lead whose side measures 44 *cm* ?
4. A hemispherical bowl of internal radius 18 *cm* contains an edible oil to be filled in cylindrical bottles of radius 3 *cm* and height 9 *cm*. How many bottles are required to empty the bowl ?
5. A hemispherical tank of radius 2.4 *m* is full of water. It is connected with a pipe which empties it at the rate of 7 litres per second. How much time will it take to empty the tank compeletely ?
6. A shuttle cock used for playing badminton has the shape of a frustum of a cone mounted on a hemisphere. The external diameter of the frustum are 5 *cm* and 2 *cm*. The height of the entire shuttle cock is 7 *cm*. Find its external surface area.
7. A fez, the headgear cap used by the trucks is shaped like the frustum of a cone. If its radius on the open side is 12 *cm* and radius at the upper base is 5 *cm* and its slant height is 15 *cm*, find the area of material used for making it. ($\pi = 3.14$)
8. A bucket is in the form of a frustum of a cone with capacity of 12308.8 cm^3 of water. The radii of the top and bottom circular ends are 20 *cm* and 12 *cm* respectively. Find the height of bucket and the cost of making it at the rate of ₹ 10 per cm^2.

9. Select a proper option (a), (b), (c) or (d) from given options and write in the box given on the right so that the statement becomes correct :

(1) The volume of sphere with diameter 1 *cm* is ...... cm^3. ☐

(a) $\frac{2}{3}\pi$ (b) $\frac{1}{6}\pi$ (c) $\frac{1}{24}\pi$ (d) $\frac{4}{3}\pi$

(2) The volume of hemisphere with radius 1.2 *cm* is ...... cm^3. ☐

(a) 1.152π (b) 0.96π (c) 2.152π (d) 3.456π

(3) The volume of sphere is $\frac{4}{3}\pi$ cm^3. Then its diameter is ...... *cm*. ☐

(a) 0.5 (b) 1 (c) 2 (d) 2.5

(4) The volume of cone with radius 2 *cm* and height 6 *cm* is ...... cm^3. ☐

(a) 8π (b) 12π (c) 14π (d) 16π

(5) The diameter of the base of cone is 10 *cm* and its slant height is 17 *cm*. Then the curved surface area of the cone is ...... cm^2. ☐

(a) 85π (b) 170π (c) 95π (d) 88π

(6) The diameter and the height of the cylinder are 14 *cm* and 10 *cm* respectively. Then the total surface area is ...... cm^2. ☐

(a) 44 (b) 308 (c) 748 (d) 1010

(7) The ratio of the radii of two cones having equal height is 2 : 3. Then, the ratio of their volumes is .... ☐

(a) 4 : 6 (b) 8 : 27 (c) 3 : 2 (d) 4 : 9

(8) If the radii of a frustum of a cone are 7 *cm* and 3 *cm* and the height is 3 *cm*, then the curved surface area is ...... cm^2. ☐

(a) 50π (b) 25π (c) 35π (d) 63π

(9) The radii of a frustum of a cone are 5 *cm* and 9 *cm* and height is 6 *cm*, then the volume is ...... cm^3. ☐

(a) 320π (b) 151π (c) 302π (d) 98π

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Summary

In this chapter we have studied the following points :

1. The surface area of some solids as under :

(1) Open cube with edge x : $5x^2$ and closed cube : $6x^2$

(2) Cylinder with radius r and height h

(a) Curve surface of a cylinder : $2\pi rh$

(b) Total surface of a cylinder : $2\pi r(r + h)$

(3) Sphere with radius r : $4\pi r^2$

(4) Hemisphere with radius r is

(a) Open hemisphere : $2\pi r^2$

(b) Closed hemisphere : $3\pi r^2$

(5) (a) Lateral surface area of cone : πrl

(b) Total surface area of cone : $\pi r(r + l)$

2. The volume :

(1) of cube with edge x is x^3

(2) of cylinder with radius r and height h is $\pi r^2 h$

(3) of sphere with radius r is $\frac{4}{3}\pi r^3$

(4) of hemisphere with radius r is $\frac{2}{3}\pi r^3$

(5) of cone is $\frac{1}{3}\pi r^2 h$

3. Conversion of solid from one shape to another.

4. For the given cone when we slice through it with a plane parallel to its base and remove the cone that is formed on one side of the plane, the portion which is left on the other side of the plane is called a frustum of a cone.

Let h be the height, l be the slant height, r_1 and r_2 are radii of ends ($r_1 > r_2$) of the frustum of a cone, then

(i) Volume of the frustum of a cone = $\frac{1}{3}\pi h\,[r_1^2 + r_2^2 + r_1 r_2]$

(ii) The curved surface area of a frustum of a cone = $\pi(r_1 + r_2) \cdot l$ where $l = \sqrt{h^2 + (r_1 - r_2)^2}$

(iii) Total surface area of a frustum of a cone = $\pi l(r_1 + r_2) + \pi r_1^2 + \pi r_2^2$

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Sharadchandra Shankar Shrikhande (born on October 19, 1917) is an Indian mathematician with distinguished and well-recognized achievements in combinatorial mathematics. He is notable for his breakthrough work along with R. C. Bose and E. T. Parker in their disproof of the famous conjecture made by Leonhard Euler dated 1782 that there do not exist two mutually orthogonal latin squares of order $4n + 2$ for every n. Shrikhande's specialty was combinatorics, and statistical designs. Shrikhande graph is used in statistical designs.

Shrikhande received a Ph.D. in the year 1950 from the University of North Carolina at Chapel Hill under the direction of R. C. Bose. Shrikhande taught at various universities in the USA and in India. Shrikhande was a professor of mathematics at Banaras Hindu University, Banaras and the founding head of the department of mathematics, University of Mumbai and the founding director of the Center of Advanced Study in Mathematics, Mumbai until he retired in 1978. He is a fellow of the Indian National Science Academy, the Indian Academy of Sciences and the Institute of Mathematical Institute, USA.